

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. SECOND SEMESTER EXAMINATION, MAY 2016

FIRST YEAR [BATCH 2015-17]

APPLIED CHEMISTRY

Date : 17/5/2016

Time : 11 am – 1 pm

Paper : VII

Full Marks : 50

[Answer a separate Answer Book for each group]

Group - A

Answer any six questions :

[6×5]

1. Find the inverse of $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & -2 & 3 \\ 1 & 2 & -3 \end{pmatrix}$ and hence solve the equations $x+2y+3z = 11$, $x-2y+3z = 3$,

$$x+2y-3z = -1.$$

[4+1]

2. Determine the conditions for which the system

$$x + y + z = 1$$

$$x + 2y - z = b$$

$$5x + 7y + az = b^2$$

admits of (i) only one solution (ii) no solution (iii) many solutions.

[5]

3. Obtain the rank of the matrix

$$\begin{pmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{pmatrix}$$

[5]

4. Obtain the characteristic roots and any one of the corresponding characteristic vectors for the matrix

$$\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

[5]

5. Verify that the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$$

satisfies its characteristic equation. Hence compute A^{-1} .

[5]

6. If the potential function is $\log \sqrt{x^2 + y^2}$, find the flux function and the complex potential function.

[5]

7. a) Prove that the function $f(z) = |z|^2$ is continuous everywhere but nowhere differentiable except at the origin.

[3]

b) Is $u = e^{-x}(x \sin y - y \cos y)$ harmonic? Justify your answer.

[2]

8. Verify Cauchy's theorem for the integral of z^3 taken over the boundary of the rectangle with vertices $-1, 1, 1+i, -1+i$.

[5]

9. Find the residues of $f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2 + 4)}$ at all its poles in the finite plane.

[5]

Group - B

Answer any four questions :

[4×5]

10. a) Show that $\int_a^b f(a+b-x)dx = \int_a^b f(x)dx$. [3]

b) Show that $\int_{a-c}^{b-c} f(x+c)dx = \int_a^b f(x)dx$. [2]

11. Express $\int_0^{\pi/2} \sin^4 \theta \cos^6 \theta d\theta$ as a Beta function and hence evaluate it. [5]

12. Find a and b in order that $\lim_{x \rightarrow 0} \frac{a \sin 2x - b \sin x}{x^3} = 1$. [5]

13. If $f(x, y) = \frac{x^2 y}{x^4 + y^2}$, discuss the existence of the limit of $f(x, y)$ as $(x, y) \rightarrow (0, 0)$. [5]

14. Prove that the function $f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}}, & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$ is continuous at $(0, 0)$. [5]

15. Show that $f_{xy}(0, 0) = f_{yx}(0, 0)$ for the function $f(x, y) = \begin{cases} \frac{x^2 y^2}{x^2 + y^2}, & \text{when } (x, y) \neq (0, 0) \\ 0 & \text{when } (x, y) = (0, 0) \end{cases}$. [5]

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